Unofficial Errata for Introduction to Tensor Analysis and the Calculus of Moving Surfaces (hardcover), Pavel Grinfeld

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Other typos found by George Katanics: PDF

Part I: Chapters 1–9

1. Page 19, line 3: "to $\mathbf{R}'(\alpha)$ and ..." should read "to $\mathbf{R}(\alpha)$ and ..."

2. Page 42, last line: **(added on 10/21/15)**
"$g(x)\frac{1}{2} = \ln x$ ..." should read "$g(x) = \frac{1}{2}\ln x$ ..."

3. Page 57, equation below (5.9): "$\mathbf{U}\cdot\mathbf{W} = ...$" should read "$\mathbf{U}\cdot\mathbf{V} = ...$"

4. Page 58, **Exercise 71**, should read "the contravariant metric tensor Z^{ij}
[Submitted by Daniel Volinski, danielvolinski@yahoo.es] **(added on 05/02/17)**

5. Page 64, last line in Fig. 5.3 caption: "$\mathbf{Z}_1 = \mathbf{i}$ and $\mathbf{Z}^1$. and $\mathbf{Z}_2 = \mathbf{j}$ and $\mathbf{Z}^2$" should read "$\mathbf{Z}_1 = \mathbf{i}$ and $\mathbf{Z}^2$. and $\mathbf{Z}_2 = \mathbf{j}$ and $\mathbf{Z}^1$"
[Submitted by Paul McCartney, pauljmccartney@gmail.com] **(added on 09/24/15)**

6. Page 66, **Exercise 76**: "in polar coordinates" should read "in cylindrical coordinates"
[Submitted by Paul McCartney, pauljmccartney@gmail.com] **(added on 09/24/15)**

7. Page 69, 1st line: **(added on 04/17/15)**
"δ^k_j. Since δ^k_j ..." should read "δ^k_i. Since δ^k_i ..."

8. Page 73, 3rd and 4th line in Sec. 5.15: "the volume element $\sqrt{Z}$." I think you meant to write:

   the length of a curve

$$L = \int_a^b \sqrt{Z_{ij}\frac{\mathrm{d}Z^i}{\mathrm{d}t}\frac{\mathrm{d}Z^j}{\mathrm{d}t}}\,\mathrm{d}t.$$

Because the volume element has not yet been introduced, see Sec. 5.8.

9. Page 89, line below equation (6.62): "is a object ..." should be "is an object ..."

10. Page 89, equation (6.64) should read

$$S_i = S^j Z_{ji}$$

11. Page 90, equation (6.65) should read

$$S^i = S_j Z^{ji}$$

12. Page 90, equation (6.69) should read

$$T^i_{\cdot j} = T_j^{\cdot i}$$

13. Page 98, equation (7.32) should read **(added on 03/24/17)**

$$(\boldsymbol{u}, \boldsymbol{v}) = M_{ij} u^i v^j$$

14. Page 102, **Exercise 112** should read $r = x - A^{-1}b$
[Submitted by Ihor Yalovetskyi , i.yalovecky@gmail.com] **(added on 05/02/17)**

15. Page 114, (8.52) should read **(added on 04/14/14)**

$$\nabla_i \mathbf{Z}_j = \frac{\partial \mathbf{Z}_j}{\partial Z^i} - \Gamma^k_{ij} \mathbf{Z}_k$$

16. Page 114, (8.53) should read **(added on 04/14/14)**

$$\frac{\partial \mathbf{Z}_j}{\partial Z^i} = \Gamma^k_{ij} \mathbf{Z}_k$$

17. Page 118, 5th line after equation (8.66): "in Sec. 8.129 ..." should be "in Sec. 8.8 ..."

18. Page 121, equation (8.79) should read

$$T^i = S^{ij} U^{kl}_j V_{kl}$$

19. Page 123, 2nd line: "we will is it ..." should read "we will use it ..."

20. Page 125, equation (8.106) should read

$$\Gamma^{k'}_{i'j'}T_{k'} = \left(\Gamma^{k}_{ij}J^{i}_{i'}J^{j}_{j'}J^{k'}_{k} + J^{k}_{i'j'}J^{k'}_{k}\right)T_{\ell}J^{\ell}_{k'} = \Gamma^{k}_{ij}J^{i}_{i'}J^{j}_{j'}T_{k} + J^{k}_{i'j'}T_{k}$$

[Submitted by Andrew Szymczak, drew.szymczak@gmail.com] **(added 06/05/17)**

21. Page 125, equation (8.107) should read

$$\nabla_{j'}T_{i'} = \frac{\partial T_{i}}{\partial Z^{j}}J^{i}_{i'}J^{j}_{j'} + J^{i}_{i'j'}T_{i} - \Gamma^{k}_{ij}J^{i}_{i'}J^{j}_{j'}T_{k} - J^{k}_{i'j'}T_{k}$$

[Submitted by Andrew Szymczak, drew.szymczak@gmail.com] **(added 06/05/17)**

22. Page 127, equation (8.124) should read

$$\Gamma^{i'}_{k'm'}T^{m'}_{j'} = \Gamma^{i}_{km}T^{m}_{j}J^{i'}_{i}J^{k}_{k'}J^{j}_{j'} + \dots$$

[Submitted by Andrew Szymczak, drew.szymczak@gmail.com] **(added 06/05/17)**

23. Page 127, equation (8.125) should read

$$\Gamma^{m'}_{j'k'}T^{i'}_{m'} = (\dots + \dots)\,T^{i}_{s}J^{i'}_{i}J^{s}_{m'}$$

[Submitted by Andrew Szymczak, drew.szymczak@gmail.com] **(added 06/05/17)**

24. Page 127, equation (8.126) should read

$$\Gamma^{m'}_{j'k'}T^{i'}_{m'} = \Gamma^{m}_{jk}T^{i}_{m}J^{j}_{j'}J^{k}_{k'}J^{i'}_{i} + \dots$$

[Submitted by Andrew Szymczak, drew.szymczak@gmail.com] **(added 06/05/17)**

25. Page 129, line above (8.132) should read **(added 5/2/17)**
"Riemann–Christoffel tensor vanishes"

26. ~~Page 129, 2nd line after (8.132) should read "the Riemann-Christoffel symbol ..."~~ **~~(added on 04/17/15)~~** Thank you George Katanics

27. Page 132, last paragraph, line 2: "properties of the covariant derivative ..." should read "properties of the derivative ..."

28. Page 137, missing the word 'indices' in the italicized definition, i.e., the definition should read "*when the upper and lower indices are ...*"

29. Page 139, line between (9.31) and (9.32) should read **(added on 04/17/15)**

$$\delta^{ijk}_{rst}a^{r}_{l}a^{s}_{m}a^{t}_{n} = A\delta^{ijk}_{lmn}$$

30. Page 141, equation (9.48) should read

$$D_1^1 = \frac{1}{2!}\left(\delta_{rst}^{123} a_1^r a_2^s a_3^t + \delta_{rst}^{132} a_1^r a_3^s a_2^t\right)$$

31. Page 144, equation (9.59) should read $\sqrt{Z} = r^2 \sin\theta$

32. Page 146, 2 lines above equation (9.72): "is a called *relative tensor* ..." should read "is called a *relative tensor* ..."

33. Page 147, 5th line below equation (9.78): "*invariant or weight 2.*" should read "*invariant of weight 2.*"

34. Page 151, 5th line after (9.97) should read **(added on 04/17/15)**
"constructing the cross product ..." instead of "constructing the gradient ..."

35. Page 157, (9.139) should read **(added on 04/17/15)**

$$(n-(k-1))\delta_{j_1\cdots j_{k-1}}^{i_1\cdots i_{k-1}} = \delta_{j_1\cdots j_{k-1} i_k}^{i_1\cdots i_{k-1} i_k}$$

Part II: Chapters 10–14

1. Page 162, equation (10.2c) should read $z(\theta,\phi) = C\cos\theta$
[Submitted by James Pedid, jamespedid@gmail.com] **(added on 05/02/17)**

2. Page 164, equation (10.11a) should be labeled as "(10.11)"

3. Page 165, 2nd line below (10.17): "the ambient and the surface covariant bases ..." should read "the ambient and the surface covariant metric tensors ..."

4. Page 166, (10.22) should read

$$Z^{i\alpha} = Z^i_\beta S^{\alpha\beta}$$

5. Page 168, (10.34) should read

$$\frac{\partial Z^{i'}}{\partial S^{\alpha'}} = \frac{\partial Z^{i'}}{\partial Z^i}\frac{\partial Z^i}{\partial S^\alpha}\frac{\partial S^\alpha}{\partial S^{\alpha'}}$$

6. Page 169, 3rd line below (10.46): "P or either 0 or 1" should read "P are either 0 or 1"

7. Page 170, **Exercise 218**: "Denote the tensor $N^i N_j$ by T^i_j" should read "Denote the tensor $Z^i_\alpha Z^\alpha_j$ by T^{ii}_j"

8. Suggestion: On page 173, move the word "Further," and (10.73) below equation (10.77). Because (10.73) is *not* needed in the derivation of (10.74) but is needed in the derivation of (10.78).

9. Page 174, 2nd line in Section 10.8: "with respect the ..." should read "with respect to the ..."

10. Page 175, (10.82) should read

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} S^{\alpha\omega} \left(\frac{\partial S_{\omega\beta}}{\partial S^{\gamma}} + \frac{\partial S_{\omega\gamma}}{\partial S^{\beta}} - \frac{\partial S_{\beta\gamma}}{\partial S^{\omega}} \right)$$

11. Page 175, (10.85) should read

$$\Gamma^{\alpha}_{\beta\gamma} = Z^{\alpha}_{i} \frac{\partial Z^{i}_{\beta}}{\partial S^{\gamma}} + \Gamma^{i}_{jk} Z^{\alpha}_{i} Z^{j}_{\beta} Z^{k}_{\gamma}$$

12. Page 177, 2nd line from the bottom: "that is. deformations" should read "that is, deformations"

13. Page 180, (10.109) should read

$$z(\theta, \phi) = r \sin \phi$$

14. Page 181, (10.113) should read

$$\Gamma^{\Phi}_{\Theta\Theta} = \frac{(R + r\cos\phi)\sin\phi}{r}$$

15. Page 181, second part of (10.118) should read

$$N^{i} = \begin{bmatrix} \frac{\cos\theta}{\sqrt{1+r'(z)^2}} \\ \frac{\sin\theta}{\sqrt{1+r'(z)^2}} \\ \frac{-r'(z)}{\sqrt{1+r'(z)^2}} \end{bmatrix}$$

16. Page 182, (10.122) should read

$$\Gamma^{Z}_{\Theta\Theta} = -\frac{r(z)r'(z)}{1 + r'(z)^2}$$

17. Page 183, in (10.131) delete extra comma after N^i

18. Page 183, 1st equation in (10.132) should read

$$Z^i_\alpha = \begin{bmatrix} 1 \\ y'(x) \end{bmatrix};$$

19. Page 183, 1st equation in (10.134) should read $\sqrt{S} = \sqrt{1 + y'(x)^2}$

20. Page 183, 2nd line from the bottom should read **(added on 11/03/14)**
"to polar coordinates (r, θ) ..." instead of "to Cartesian coordinates (r, θ) ..."

21. Page 189, (11.14) should read

$$\mathbf{S}^\delta \cdot \nabla_\alpha \mathbf{S}_\beta = \mathbf{S}^\delta \cdot \frac{\partial \mathbf{S}_\beta}{\partial S^\alpha} - \Gamma^\delta_{\alpha\beta}$$

22. Page 190, "*Example 237*" should read "**Exercise 237**"

23. Page 190, (11.21) should read

$$\nabla_\gamma \mathbf{T} = \frac{\partial \mathbf{T}}{\partial S^\gamma} = \frac{\partial \left(T^i \mathbf{Z}_i\right)}{\partial S^\gamma} = \frac{\partial T^i}{\partial S^\gamma}\mathbf{Z}_i + T^i \frac{\partial \mathbf{Z}_i}{\partial S^\gamma}$$

24. Page 190, (11.22) should read

$$\nabla_\gamma \mathbf{T} = \frac{\partial T^i}{\partial S^\gamma}\mathbf{Z}_i + T^i \frac{\partial \mathbf{Z}_i}{\partial Z^j}\frac{\partial Z^j}{\partial S^\gamma}$$

25. Page 190, (11.23) should read

$$\nabla_\gamma \mathbf{T} = \frac{\partial T^i}{\partial S^\gamma}\mathbf{Z}_i + T^i Z^j_\gamma \Gamma^k_{ij}\mathbf{Z}_k$$

26. Page 190, (11.24) should read

$$\nabla_\gamma \mathbf{T} = \left(\frac{\partial T^i}{\partial S^\gamma} + Z^k_\gamma \Gamma^i_{km} T^m\right)\mathbf{Z}_i$$

27. Page 191, (11.25) should read

$$\nabla_\gamma T^i\, \mathbf{Z}_i = \left(\frac{\partial T^i}{\partial S^\gamma} + Z^k_\gamma \Gamma^i_{km} T^m\right)\mathbf{Z}_i$$

28. Page 191, (11.26) should read

$$\nabla_\gamma T^i = \frac{\partial T^i}{\partial S^\gamma} + Z^k_\gamma \Gamma^i_{km} T^m$$

29. Page 191, (11.27) should read

$$\nabla_\gamma T^i_j = \frac{\partial T^i_j}{\partial S^\gamma} + Z^k_\gamma \Gamma^i_{km} T^m_j - Z^k_\gamma \Gamma^m_{kj} T^i_m$$

30. Page 191, (11.28) should read

$$\nabla_\gamma T^{i\alpha}_{j\beta} = \frac{\partial T^{i\alpha}_{j\beta}}{\partial S^\gamma} + Z^k_\gamma \Gamma^i_{km} T^{m\alpha}_{j\beta} - Z^k_\gamma \Gamma^m_{kj} T^{i\alpha}_{m\beta} + \Gamma^\alpha_{\gamma\omega} T^{i\omega}_{j\beta} - \Gamma^\omega_{\gamma\beta} T^{i\alpha}_{j\omega}$$

31. Page 192, (11.32) should read

$$\nabla_\gamma T^i_j = \frac{\partial T^i_j}{\partial S^\gamma} + Z^k_\gamma \Gamma^i_{km} T^m_j - Z^k_\gamma \Gamma^m_{kj} T^i_m$$

32. Suggestion: Move **Exercise 253** on page 202 to the *end* of **Section 12.2**.

33. Page 203, (12.22) should read

$$\left(\nabla_\alpha \nabla_\beta T^\gamma_\delta - \nabla_\beta \nabla_\alpha T^\gamma_\delta\right) S^\delta = R^\gamma_{\cdot\omega\alpha\beta} T^\omega - R^\delta_{\cdot\omega\alpha\beta} S^\omega T^\gamma_\delta$$

34. Page 203, (12.23) should read

$$\left(\nabla_\alpha \nabla_\beta T^\gamma_\delta - \nabla_\beta \nabla_\alpha T^\gamma_\delta\right) S^\delta = R^\gamma_{\cdot\omega\alpha\beta} T^\omega_\delta S^\delta - R^\omega_{\cdot\delta\alpha\beta} T^\gamma_\omega S^\delta$$

35. On page 207, (12.50) is only correct if one chooses $S^1 = \phi$ and $S^2 = r$ (with the normal away from the z-axis). If one chooses $S^1 = r$ and $S^2 = \phi$, then the $1,1$ element in all of the matrices in (12.50) becomes the $2,2$ element.

36. Page 208, (12.59) should read

$$B^\alpha_\alpha = \frac{r''(z)r(z) - r'(z)^2 - 1}{r(z)(1 + r'(z)^2)^{3/2}}$$

37. Page 212, delete period on the line right above (12.85).

38. Page 212, (12.85) should read

$$C_{\alpha\beta} = B^{\omega}_{\omega} B_{\alpha\beta} - K S_{\alpha\beta}$$

39. Page 187, line between (11.5) and (11.6): "The Laplacian ..." should read "The surface Laplacian ..."

40. Page 192, it would be better to say "The object $\partial Z^k / \partial S^\gamma$ is ... Z^k_γ..." on the line below (11.33).

41. Page 199, 2nd paragraph, 3rd line: "in the perspective in the Riemann–Christoffel ..." should read "in the perspective is the Riemann–Christoffel ..."

42. Page 217, 1st line: "Z^i and that the embedded" should read "Z^i and the embedded ..."

43. Page 217, (13.15) should read

$$s = \int_a^b \sqrt{U}\, \mathrm{d}U^1.$$

44. Page 221, line above (13.34): "a product its length κ ..." should read "a product of its length κ ..."

45. Page 221, (13.40) should read

$$B^\alpha_\alpha = \frac{\sin t}{(1 + \cos^2 t)^{3/2}}.$$

46. Page 222, 1st line in the Fig. 13.1 caption: "The first two plots ..." should read "The top two plots ..." Also, add a period at the end of the caption.

47. Page 224, (13.53) should read

$$\frac{\mathrm{d}\mathbf{Q}}{\mathrm{d}s} = \kappa \mathbf{P} \times \mathbf{P} + \mathbf{T} \times (-\kappa \mathbf{T} + \tau \mathbf{Q}) .$$

48. Page 228, line between (10.3) and (13.87): **(added on 10/21/15)**
"the embedded surface ..." should read "the embedded curve ..."

49. Page 229, line below (13.18): "Christoffel symbol $\Gamma^\alpha_{\beta\Gamma}$ ..." should read "Christoffel symbol $\Gamma^\alpha_{\beta\gamma}$ ..."

50. Page 229, (13.93) should read

$$\Gamma^{\Omega}_{\Phi\Psi} = \Gamma^{\alpha}_{\beta\gamma} S^{\Omega}_{\alpha} S^{\beta}_{\Phi} S^{\gamma}_{\Psi} + \frac{\partial S^{\alpha}_{\Phi}}{\partial U^{\Psi}} S^{\Omega}_{\alpha}.$$

51. Page 229, line between (13.100) and (13.101): **(added on 10/21/15)**
"We next the curvature ..." should read "We define the curvature ..."

52. Page 231, line below (13.113): "By equation (10.31) ..." should read "By equation (10.55) ..."

53. Page 237, 3rd sentence in the caption of Fig. 14.1: "The firgure includes ..." should read "The figure includes ..." Also, add a period at the end of the caption.

54. Page 240, (14.16) should read **(added on 10/22/15)**

$$\int_{A_3}^{B_3} \int_{A_2}^{B_2} \int_{A_1}^{B_1} F(Z)\, \mathrm{d}Z^1 \mathrm{d}Z^2 \mathrm{d}Z^3 = \int_{A_{3'}}^{B_{3'}} \int_{A_{2'}}^{B_{2'}} \int_{A_{1'}}^{B_{1'}} F(Z')|J|\, \mathrm{d}Z^{1'} \mathrm{d}Z^{2'} \mathrm{d}Z^{3'}$$

55. Page 241, (14.17) should read **(added on 10/22/15)**

$$= \int_{A_{3'}}^{B_{3'}} \int_{A_{2'}}^{B_{2'}} \int_{A_{1'}}^{B_{1'}} F(Z(Z'))\sqrt{Z}|J|\, \mathrm{d}Z^{1'} \mathrm{d}Z^{2'} \mathrm{d}Z^{3'}$$

Note that by $\sqrt{Z}$ we really mean $\sqrt{Z} = \sqrt{|Z_{..}|} = \sqrt{|Z_{..}(Z')|}$, i.e., the determinate of Z_{ij} is expressed in the prime coordinates. For example, when switching *from* polar coordinates *to* Cartesian coordinates we must write $\sqrt{Z}$ as $\sqrt{Z} = \sqrt{x^2+y^2}$ and not as $\sqrt{Z} = r$, then $\sqrt{Z}|J| = \sqrt{x^2+y^2}\frac{1}{\sqrt{x^2+y^2}}$.

56. Page 241, line between (14.17) and (14.18): **(added on 10/21/15)**
"Sect. 9.9 that ... invariant of weight -1" should read "Sect. 9.9 (**Exercise 194**) that ... invariant of weight 1"

57. Page 241, (14.18) should read **(added on 10/22/15)**

$$\sqrt{Z'} = J\sqrt{Z}$$

58. Page 242, line between (14.24) and (14.25): **(updated on 10/22/15)**
"Sincev" should read "Since"

59. Page 244, (14.43) should read

$$T_i = \varepsilon_{ijk} N^j n^{\alpha} Z^k_{\alpha}$$

(Note the capital Z instead of z.)

60. Page 245, (14.52) should read

$$\int_U F^i T_i \, \mathrm{d}U = \int_S \nabla^m F^i \varepsilon_{ijk} N^j \delta^k_m \, \mathrm{d}S - \int_S \nabla^m F^i \varepsilon_{ijk} N^j N^k N_m \, \mathrm{d}S.$$

Bibliography

[23] F. Harley should read H. Flanders
[Submitted by Don Benson, dbenson9@shaw.ca] **(added on 05/03/17)**

Index

1. Shift tensor . . . . . 164 **(added on 01/09/17)**